Farook Training College Innovative Academia (FTCIA)

Online Collaborative Learning Project (OCLP)

Pre-Edited Version of Study Materials.

(Chance for minor errors)

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The entire materials are prepared by the B.Ed students (2020-22) of Farook Training College, Calicut, Kerala.

It is expected that this will be a support for those who need simplified, concise but comprehensive study materials for their examination preparation. It is a smart footstep to self learning and peer learning.

A note of appreciation to all student teachers who are the workforce behind this great endeavor.

Team OCLP

FTC

B Ed. I. Sem. EDU.05.10 THEORETICAL BASES OF TEACHING MATHEMATICS

Unit 1

Nature of Mathematics

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MATHEMATICS – Meaning and Definition

- The term mathematics is said to be traced from **mendh** meaning **to learn**, related to the Greek word **manthenein**.
- The Latin origin is **ars Mathematica**, the Greek origin **mathematike techne**, meaning **mathematics as an art and mathematics as a science**.
- Mathematics is the science of logical reasoning, science of quantity and space.
- It is a science, a language, an art, a game, and a tool.

- Mathematics is :

i. a systematized, organized and abstract form of science
ii. the science of calculations
iii. the science of logical reasoning
iv. the science which helps to draw necessary conclusions
v. an inductive and experimental science

NATURE OF MATHEMATICS

- It has its own language which is well defined, clear and useful.
- Mathematics helps in the development of scientific attitude among children.
- Mathematics gives accurate and reliable knowledge.
- It develops critical thinking.
- Mathematical knowledge is exact, systematic, logical and clear.
- Mathematical rules, laws and formulae are universal and well accepted that can be verified at any place and time.
- It develops the ability of induction and deduction.

Mathematics as a Science:

- It is the science in which numbers are of prime importance.
- Mathematics has its origin in numbers and hence number system is a special field by which other branches of mathematics are developed.
- Mathematics is considered as the numerical and calculation part of man's life and knowledge.

Mathematics as a Game:

- In mathematics, we create a set of rules and regulations just like in a game, and proceed with logical reasoning.
- For many mathematicians, matgematics is a game with figures and numbers.

Mathematics as a Language:

- As a language , mathematics is characterized by symbols to express size, shape and order.
- These symbols make mathematical expressions precise, concise and accurate.

Mathematics as a Tool:

- Mathematics is a tool in the sense that it contains the skills and competencies for organizing, simplifying, calculating and interpreting numerical data.
- This is essential for learning and solving problems of Sciences, Social Science, Business, Industry and the like.

CHARACTERISTICS OF MATHEMATICS

- Objectivity and predictability
- Free from social influences (universal)
- Self contained
- Interconnected structures
- Simplicity and accuracy of reasoning
- Originality of thinking
- Similarity to the reasoning of daily life
- Certainity of results

Difference between Mathematical Science and Basic Science

- Mathematics is science but it differs from any basic science. In science truth means correspondence with observation whereas in mathematics truth is the logical consistency of the statements made and the results obtained.
- Basic science is experimental in nature but mathematics is progressive in its nature.
- Basic science gives importance to observation whereas mathematics gives importance to logic.

PURE AND APPLIED MATHEMATICS

- Mathematics as a discipline focuses on two significant aspects, development of the subject, I.e., adding more to the existing knowledge and application of the known ideas in various walks of life, occupation and other disciplines. Thus we can consider two types of mathematics – pure and applied.

(1) PURE MATHEMATICS:

- It treats theories and principles without considering their application in the real life.
- It involves systematic and deductive reasoning and is abstract and self contained in nature.
- It is confined to the theoretical development of the subject.
- Eg: Number theory, Algebra, etc.

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(2) **APPLIED MATHEMATICS**:

- It focuses on the application of mathematical principles and concepts in different fields.
- It helps in the development of other sciences/subjects.
- It can be considered as a link between the Pure Mathematics and the various disciplines like physical, biological and social sciences, technology, etc...
- Eg: Probability theory, cryptography, etc...

AXIOMS AND POSTULATES

AXIOMS:

- Every area in mathematics is based on some statements which are assumed to be true and not proved. These are self-evident truths which we take to be true without proof. These statements are called axioms.
- “Axiom” comes from the Greek word "axioma", which mean to "to deem worth".
- Axioms are background assumptions we make and is a premise or starting point of reasoning.
- The statements we choose as axioms require a lot of thought and insight. Axioms serve as basis for deducing other truths.

- Examples:

a. Things which are equal to the same thing are equal to one another.
b. If equals are added to equals, the wholes are equal.
c. If equals are subtracted from equals, the remainders are equal.

POSTULATE:

- Non- logical axioms usually called postulates, define properties for the domain of a specific mathematical theory, or logical statements, which are used in deduction to build mathematical theories.
- Postulate is actually a verb which means to 'demand'.
- Postulates aims to capture what is special about a particular structure.
- Examples:

a. A straight line may be drawn from one point to any other point.
b. A terminated line can be produced indefinitely.
c. A circle can be drawn with any centre and any radius.

How are they differentiated?

- Axioms are self evident assumptions which are common to all branches of science, while postulates are related to a particular science.
- Unlike axioms, postulates aim to capture what is special about a particular structure.
- Axioms are merely background assumptions we make while postulates are facts based on reasoning.

FUNDAMENTAL BRANCHES OF MATHEMATICS

- Mathematics as a discipline has four major branches. They are:

1. Arithmetic
2. Algebra
3. Geometry
4. Trigonometry

1. ARITHMETIC

- The term ‘arithmetic’ is derived from the Greek word ‘arethmetike’ which means the science of number.
- Arithmetic is the science of number and art of computation.
- It is the branch of mathematics concerned with numerical calculations, such as addition, subtraction, multiplication, and division.
- It involves calculations involving numerical operations.
- Historically arithmetic developed out of the need for a system of counting.
- The teaching of arithmetic has two major responsibilities.

  (a) The building up of thorough understanding of the number system and proficiency in the fundamental processes.

  (b) Making use of this knowledge for the improvement of humanity.

2. ALGEBRA

- The term ‘algebra’ is derived from the Arab word ‘Al-jebr’ which means joining of broken parts.
- Algebra is a method of calculating, using letters, representing numbers and is a science to show relations between numbers and letters making a kind of abstract arithmetic. I.e., Algebra is a generalisation of arithmetic.
- It is that branch of mathematics in which symbols, usually letters of the alphabet, represent numbers or members of a specified set and are used to represent quantities and to express general relationships that hold fo all members of the set.

3. GEOMETRY

- The word ‘geometry’ is derived from the combination of two Greek words ‘geo’ (means earth) and ‘metron’ (means measure).
- The word geometry literally means ‘to measure land.’
- Geometry is the mathematics of properties, measurement, relationships of points, lines, angles, surfaces and solids.
- It is the science of properties and relations of magnitudes in space as lines, surfaces, etc...

4. TRIGONOMETRY

- The word trigonometry has a Greek origin namely ‘trigonon’ means triangle.
- Trigonometry is the branch of mathematics that deals with the relationships between the sides and angles of triangles and calculations based on them, particularly the trigonometric functions.
- It is the branch of mathematics dealing with the relations of the sides and angles of triangles which is used in surveying, navigation, etc...
- Trigonometry can be considered as a combination of arithmetic, algebra and geometry.

RELATIONSHIPS WITHIN THE BRANCHES OF MATHEMATICS

- The different branches of mathematics namely, arithmetic, algebra, geometry and trigonometry are interrelated.
- These different branches are water-tight compartments.
- Algebra is often referred to as generalized arithmetic and also it can be treated as written geometry whereas geometry is referred to as pictured algebra.
- Integrated approach recommended by Indian Education Commission, 1964-66

CORRELATION OF MATHEMATICS WITH OTHER SUBJECTS

- All the engineering disciplines need mathematics for solving problems. Areas like differential equations in calculus, probability theory, combinatorics and algebraic geometry are some major areas of mathematics which have immense application in the field of engineering.
- Mathematics is the gateway and key to all physical sciences. Physics and chemistry cannot exist without mathematical ideas and language. Principles and relations in these subjects are expressed precisely using mathematical language and symbols.
- Biology and medicine have much benefits from the intervention of sophisticated mathematical tools. Topology, algebra and calculus are found to be much applied in the field of biological sciences.

- Mathematics has its application in social science also. Economics is a mathematics based subject. Probability and statistical techniques are used to analyze the social and psychological phenomena. Mathematics is used very much while studying the laws of population change in ecology.
- Computer science is product of mathematics, with logic and combinatorics as the basic. The programmes that are run even in the super computers are based on mathematical principles.
- Logic and philosophy are also related with mathematics. Mathematics is the only field of knowledge in which the logics are applied and the study of which develop logical reasoning. To know the validity of the truth and reality in philosophy, mathematical ideas are used.

- Agriculture as a science has direct application of mathematics at various aspects like measurement of area of land, average investment/income, production per unit area, cost of labour, time and work, seed rate, manure rate, etc... Graphs and diagrams can also be used for expressing and judging production of various crops in farms.
- Behavioural sciences cannot attain its present status without the application of mathematical knowledge. Psychology especially experimental psychology uses mathematics and statistics for the experimentation, collection of data, its analysis and interpretation of the results.

RELATION OF MATHEMATICS WITH DAILY LIFE

- Mathematics is an indispensable part of our daily life.
- One starts a day by using mathematics when he adjusts his time for various activities, so that he can join his duty at the right time.
- While calculating wages, planning the expenses, estimating the balance, buying or selling things, doing banking, etc... mathematics is used in one or another form.
- The taste of food depends on the appropriate quantity or ratio of the ingredients it needs.
- An individual to whatever field he belongs- agriculture, carpentry, tailoring, business, banking, share market- use the knowledge of mathematics at each and every second of his life.
- Mathematics is in every sphere of life.

DEVELOPMENT OF MATHEMATICS AS A SCIENCE

- Mathematics is a man-made science. It is a progressive science which has acquired the qualities of an abstract science. The first stage of development of the subject can be located as the use of numbers and geometrical patterns for the practical purposes. Later intuition and logical thinking made the subject more developed and abstract in nature. Thus, the development of mathematics as a science can be conceived as of three stages.

i. Empirical
ii. Intuitional
iii. Rational

i) **Empirical Stage**:

- As any invention, the mathematical knowledge was first used to solve the problems of dealing with quantitative data. In the history of human civilisation, when man has started to live in groups, started to hunt, started raising cattle, started agriculture, man was in need of counting, measuring or drawings. This was the starting of Mathematics.
- The man used small sticks/ stones / bones of animals to get the number of animals, number of food items procured, weapons they have etc. As needs expanded the use of maths was also improved. this stage is known as the empirical stage.

ii) **Intuitional Stage**:

- The mental images formed during the direct experience with objects helped the men to think more about the ideas and thus they came into new concepts/ideas.
- The idea of marking the numbers / symbols/geometrical figures on bones and rocks can be considered as a result of this intuition. This stage is known as intuitional period.

iii) <u>Rational Stage</u>:

- Reasoning in thinking, the inductive and deductive thinking made the men analyze the problematic situations, establish the possible relationship between various ideas and solve the problems they confront. the logical thinking and reasoning made the subject more scientific and abstract.

HISTORY OF MATHEMATICS

History of mathematics is the **history of mankind**

This history can be classified into 8 periods :

1.Period of Ancient, Egyptians and Babylonians

2.Greek and Roman period (600 B.C to 500 A.D)

3.Middle Ages (500 A. D to 1440 A.D)

4.Renaissance period (15th and 16th century)

5.The 17th century

6.The 18th century

7.The 19th century

8.The 20th century

1.Period of Ancient, Egyptians and Babylonians

Before 4000 B.C (No recorded historical data)

- Dates back to 30,000 B.C
- Activities focused on **numbering**
- People might have worked at **concrete level**, that is, doing things like :

~Bundling sticks in one to one correspondence with fingers

~Made tallies on bones and sticks

~Arranged pebbles in blocks, arrays and piles

~Tied knots in strings

~Created geometric patterns in baskets

About 4000 B.C

- Developed an **elementary number system** like “1,2 and many” (1,2,3,4...came later). The concept of ‘zero’ and ‘positional notation’ didn't exist then.
- Developed a **calendar**
- Developed a **system of weights and measures**

Egyptians

- It is said, **geometry** originated in Egypt
- Represented numbers by succession of strokes
- Used fractions

- Ahmes(their earliest mathematician, 1700 B.C) developed: arithmetic, and an inaccurate geometry which lacked in symbolism

Babylonians

- Used **positional notation** by about 2500 B.C
- Recognised **zero** as a number; invented a symbol for zero (between 300 B.C and 700 A.D)
- Had knowledge of **reciprocals**
- Used **cuneiform** (wedge shaped characters) to represent numbers
- Constructed **mathematical tables** of :Multiplication, squares, cubes and reciprocals.

- Developed **algebra**
- Formulated rules for finding **areas** of: squares, rectangles, right angled triangles.
- They were ***mathematically superior to Egyptians*** esp in algebra and computation.

2.Greek and Roman period (600 B.C to 500 A.D)

Thales, Pythagoras, Plato, Archimedes, Eratosthenes, Hipparchus and Ptolemy were the prominent mathematicians of this period

Thales

- Began proving theorems. Founded deductive proof. He emphasised on proof rather than intuition in geometry
- Founded the earliest Greek school of mathematics and philosophy

Pythagoras

- Founded the Pythagorean School of mathematics and other schools

- Raised mathematics to the field of science
- First to formulate the proof of Pythagorean theorem
- **Pythagoreans**

They credited Pythagoras with their new discoveries

1. Founded the Pythagorean theorem
2. Identified root 2 as an irrational number
3. The first to discover that earth is a sphere
4. Contributed to number theory, geometry and algebra
5. Used deductive approach in geometric proofs

Plato

- Made the rule that, only a ruler and a pair of compasses are needed for geometric construction

Euclid

- Euclidean Geometry
- Deductive proof of theorem

Archimedes

- Worked in the field of geometry and arithmetic
- Principle of buoyancy
- Discoveries regarding sphere and cylinder
- ‘Method of exhaustion’ brought to full maturity
- Results regarding volume and surface area of sphere; area of a parabolic segment

Eratosthenes

- Worked in number theory and geography
- Calculation of circumference of earth

Hipparchus

- Began the study trigonometry and emphasised the use of similar triangles in trigonometry
- Discovered the sun's annual orbit is eccentric

Ptolemy

- Contributions to trigonometry and astronomy
- Introduced the idea that earth as the centre of the universe;concept of epicycles.

3.Middle Ages (500 A.D to 1440A.D)

- A period of stagnation in development of mathematics, but remarkable contributions made by the **Hindus and Arabs.** Oriental(Asian) mathematics originated as a **practical science** to facilitate : computation of calendar, organisation of public works, collection of taxes etc.

Contributions of Hindus

- In arithmetic, concept of **zero, negative numbers, decimal place notation**
- Mathematicians like Aryabhatta, Bhaskaracharya, Brahmagupta etc.

Contribution of Arabs

- To **trigonometry**
- Familiar **with geometric solution of quadratic and cubic** equations
- **Translated** Euclid's 'Elements' and Ptolemy's 'Almagest' **to Arabic.**

Most outstanding mathematician of this age - **Leonardo Fibonacci**. He translated 'Elements' and 'Almagest' from Arabic to Latin. Published the books: 'Liber Abaci', 'Practica Geometrica'.

4.Renaissance period (15th and 16th century)

- **Algebra and trigonometry** got attention
- Johannes Muller made trigonometry a subject independent of astronomy
- Copernicus dealt with trigonometry and astronomy. He claimed that sun and planets do not revolve around earth but the earth and other planets revolve around sun.
- John Napier made significant contributions

5.The 17th Century

John Napier

- Introduced algebraic concept of logarithms
- Invented a mechanical computer that used rods for mathematical computations

Galileo Galilei

Founder of dynamics (the study of motion of objects)

Johannes kepler

- Propounded three laws of planetary motion

Fermat

- Founded the branch of number theory

John Graunt

- Founded the field of statistics

Blaise Pascal

- Invented a mechanical adding machine which contained cogwheels

Projective Geometry by :Desargues and Blaise Pascal

Analytic Geometry by :Rene Descartes and Fermat

Probability by : Fermat and Pascal

Calculus by :Isaac Newton and Wilhelm Leibniz

Mathematical Statistics originated in this period

6.The 18th century

- Mathematicians were less secretive then earlier mathematicians.
- Calculus further developed and refined

Leonhard Euler

- The most outstanding mathematician of that time
- Contributed to all the branches of mathematics known at that time
- Evolved network theory(a branch of topology)

Louis Lagrange, Simon Laplace, James Bernoulli, John Bernoulli, Abraham de Moivre developed existing branches of mathematics

- Developed new branches like infinite series, Differential Geometry, calculus of variation and descriptive geometry

7.The 19th century

- *The golden era of modern mathematics*
- Advancements in both mathematics and Science.
- Further development and refinement of existing branches (Probability, topology and calculus)
- creation of new branches like set theory and abstract algebra
- These paved way for all developments of modern mathematics

- New emphasis on abstract or pure mathematics
- Invention of electricity gave way to computers and mathematics a new dimension
- Mathematicians of this period established that mathematics is the science of logical structures

Charles Babbage

- Attempted to construct a calculating machine (useful for construction of mathematical tables)

Herman Hollerith

- Devised a way of recording sensors data on punched cards.

- In 1896 started the Tabulating machine company (later became International Business Machines)

Evaristle Galois

- Introduced the concept of **groups** in higher algebra

George Cantor

- Rapid growth of topology by creation of **set theory**

George Boole

- Founded **Boolean Algebra**
- developed a system of symbolic logic; published a book on algebra of logic

Carl Friedrich Gauss

- Constructed a system of geometry different from Euclid's (it showed that it is possible to have two or more valid mathematical systems which contradict each other)

Several other mathematicians came up with a geometry different from Euclid.

8. The 20th century

- Abstract algebra, Calculus and Topology matured
- The first totally automatic electromechanical computer presented

- Theory of relativity by Albert Einstein
- New areas of mathematics like computability theory, complexity theory, recursive function theory, operations research, linear programming and Game Theory developed.
- An increase in the importance of discrete concepts and expansion of combinatorics including graph theory

Ramanujan

- Made investigations in areas of: gamma function, modular forms, divergent series, hypergeometric series and prime number theory

Importance of history of mathematics

- Gives a detailed description of main trends in the development of mathematics throughout the ages
- for proper understanding of the subject
- getting knowledge about all happenings in the area of mathematics and a time sequence
- Knowing the practical value of mathematics in daily life
- for developing interest in mathematics study
- having an idea about history of Civilization
- knowing the interdependence of the subject
- Developing appreciation of contributions of mathematician

Contributions of some great mathematicians

ARYABHATTA (476 A.D to 550 A.D)

- Wrote the book Aryabhatiya (an astronomical treatise)
- First to introduce the concept of zero and a symbol for it
- Numeration by tens
- Value of pi (3.1416)
- Method for square root and cube root
- Solution of quadratic equations
- Rule for summation of arithmetic series
- Area of an isosceles triangle
- Rule for finding sines and a table of these functions

Aryabhatiya

- Has **118 verses** giving summary of Hindu mathematics
- The system of denoting digits by letters, arithmetic , algebra ,astronomy, geometry, trigonometry are dealt with
- It has 66 mathematical rules without proof
- Has verses on: reckoning of time and planetary models; on sphere and eclipses.

Bhaskaracharya ll (1114-1185)

- Wrote Lilavati, Beejaganita(on algebra), siddhantha siromani(on astronomy)
- Treatment of negative numbers as debts or losses
- worked on series, permutation, linear and quadratic equations, mensuration and astronomy.
- concept of infinity
- proof for Pythagoras Theorem
- solution of determinate and indeterminate problems

Brahmagupta (598 - 668 A.D)

- Major work is a book on astronomy - Brahma Siddhanta (including arithmetic and indeterminate equations)
- 3 as practical value of pi and root 10 as the correct value
- Exact concept of zero
- rules for negative numbers
- worked on integers and fractions, progressions, rule of Three, Mensuration of plane figures, simple interest, problems on volumes and on Shadow reckoning
- 4 methods of multiplication
- Explanation for method of inversion

- Solution of quadratic equation
- solution of indeterminate equations
- two sets of values for the sides of a right angled triangle

Sreenivasa Ramanujan (1887-1920)

- Theory of numbers
- every even integer greater than 2 is the sum of two prime numbers
- hypergeometric series and continued fractions
- Worked on definite integrals and elliptic functions
- composite number and divergent series
- formula for computing the exact value of the number of partitions of any whole number

Pythagoras (570-495 B.C)

- He learnt mathematics from Thales(the founder of Greek mathematics)
- founded a school (members known as Pythagoreans); motto of this school was “all is number”
- he explained the musical harmony in terms of whole number ratios (harmonic proportion)
- theory of irrational qualities
- construction of mundane figures
- proposition related to the sum of angles of a triangle
- Definition of points as ‘unity having position’

- Theorem relating to the square of the hypotenuse of a right angled triangle
- generation of Pythagorean triplet

Euclid (300 B.C)

- The book Elements containing geometric and arithmetic principles and theorems that formed the basis of mathematics for more than 2000 years. Geometry here deals with: Triangles, congruence, parallels, areas of rectilinear figures, geometric algebra, circles,chords, tangents, inscribed and circumscribed polygons etc.
- LCM and GCF

- Book ‘Phenomena’ dealing with celestial sphere and containing 25 geometric propositions
- ‘Data Cataprica’ a treatise on music
- interesting account of theory of numbers
- introduction of prime and composite numbers
- prime numbers are infinite

2 Mark Questions

1. Write any two peculiarities of Mathematics language.
2. Write any two examples of correlation of algebra with geometry.
3. Write two examples for correlation with life.
4. How applied mathematics is related to pure mathematics?
5. How you explain mathematics as a game?

4 Mark Short Essay

1. Briefly explain the development of Mathematics.
2. Mathematics is a science, but it is different from other sciences. Explain.
3. Which are the fundamental branches of mathematics? What are the link between them?
4. How Mathematics act as a tool for learning other school subjects? Illustrate.

10 Mark Essay

1. Briefly explain the development of Mathematics from Vedic period to 20^{th} century.

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Unit 2

Aims and Objectives Teaching Mathematics

GROUP MEMBERS

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Values of learning Mathematics

a) <u>Practical or Utilitarian Values</u>

- Person ignorant of Mathematics will be at the mercy of others and will be easily cheated.
- Preliminary requirements of human being.
- In many occupations indirect or direct use of Mathematics is made.
- A person with proper calculations, can anticipate all the possible handicaps to be faced and thus can adopt precautionary measures.
- Mathematical illiteracy in the masses is a formidable barrier in the way of a country's progress.

b) <u>Disciplinary Values</u>

- Strengthening of mental power as well as acquisition of knowledge is considered as an important disciplinary value of Mathematics.
- Acquisition of the art of proper thinking and effective reasoning.
- Imbibe in the individual values like honesty, open-mindedness, etc.
- Helps an individual to lead a well-disciplined life, which in turn helps him to constitute for a good Civil Society.
- Makes the mind of the learner more broad and open.
- Helps to analyse the situation and to perceive correctly the state of affairs.
- Makes the mind of the is thoughts more accurately and explicitly.

c) Cultural Values

- The changes in modes of living and the culture has been continuously influenced by the progress in Mathematics.
- ‘Mathematics is the mirror of civilization’.
- The history of Mathematics portrays the culture and civilization of a Civil Society.
- Helps in the preservation and transmission of our cultural traditions.

d) Social Values

- Helps in the proper organization and maintenance of a fruitful social structure.
- Plays an important role in the proper setting up of social institutions such as banks, co-operatives, railways, etc
- Effective business transactions, exports and imports, trade and commerce and communication cannot take place without Mathematics
- Smooth and orderly functioning of the civil society is ensured by Mathematics.
- Our social existence is totally governed by the scientific and technological knowledge, which can only be attained by the study of Mathematics.
- Help the individual to adjust himself and lead a harmonious life in the society.

e) Aesthetic Values

- The elegance and gracefulness of Mathematical relationships touches our emotions, much like music and art can reach inside the psyche and make us feel truly alive.
- The fineness, the harmony, the symmetry everything adds its beauty.
- Music or art are simply the aftermath of this eternal beauty.
- Biographies of great Mathematicians tells us that almost all of them were attracted to this ‘divine discipline’, by realising its beauty.

f) <u>Vocational Values</u>

- Study of mathematics prepares the students for a wide variety of vocations
- It finds extensive application in all vocations like:
- Agriculture • Accountancy • Banking • Business • Engineering • information technology • Tailoring • Carpentry • surveying etc.

g) <u>Moral Values</u>

- Helps an individual in his character formation
- It develops in him a proper attitude, as there is no space for prejudiced feelings, biased outlook, discrimination and irrational thinking, and aids him in objective analysis, correct reasoning, valid conclusions and impartial judgment his character formation

h) Recreational Values

- Mathematics gives a variety of recreational opportunities to grown up people as well as children
- Various puzzles, games, riddles, etc., of Mathematics, give people recreation and entertainment.
- The study of Mathematics thus gives sufficient exercise to the brain of an individual.

Aims of teaching Mathematics

AIMS

- Aims are the distant goals of education.
- Any system of education focuses on realisation of certain aims, which give direction for the entire educational process.
- The entire programmes of the school ar determined by these aims
- Each subject of study has its own aims, but derived from the general aims of education.

Aims of teaching mathematics

1) **Practical/ Utilitarian aims**

- The aims based on the applicability of mathematics in daily life of an individual

2) **Disciplinary aims**

- Aims for developing reasoning ability that is capable of training the mind which an individual need for effective living

3) **Cultural and social aims**

- Know about the cultural heritage and the role of mathematics in dealing problems faced during the social development.

1) **Practical/ Utilitarian aims**

- 7 R's – Reading, Writing, Arithmetic, Rights, Responsibilities, Recreation and Relationship.
- The aims based on the applicability of mathematics in daily life of an individual
- *Enabling the child to solve mathematical problems of his daily life.*
- *Preparing him for elementary as well as higher education in various disciplines.*
- *Providing the basis of mathematical skills and processes which will be needed for vocational purposes.*

2) Disciplinary aims

- Providing suitable type of discipline to the mind of the learner.
- Developing the habits of concentration, self-reliance and discovery.
- Creating in the child love for hard work.
- Developing in the child the power of thinking & reasoning.
- Developing learners' power of expression.
- Helping the individual become self-reliant and independent.

3) Social and cultural aims

- Developing the child an acquaintance with his culture
- Developing a sense of appreciation of cultural arts.
- Enabling the child understand and enjoy popular literature.
- Developing a scientific and realistic attitude towards life.
- Preparing the child for economic, productive, creative, purposeful and constructive learning.

Objectives of teaching Mathematics

Objectives

- Aims are broad and general in nature
- a teacher can not make them achieved by learners within a limited duration
- attainment of general aims is not easily verifiable.
- More specific, clear and verifiable goals are needed which are attainable through planned activities within a fixed period.
- The teaching learning process planned according to these goals , the attainment of which can be verified systematically.
- A teacher has specific objectives of teaching, the instructional objectives, which can be realized through planned definite activities and be verified through different techniques.

Instructional Objective

- a statement that describes what the pupil will do/ be able to do, after the learning period, ultimately leading to the realization of the educational aims.
- help the classroom teacher to design the instruction, use appropriate techniques of evaluation and
- Help students know about their expected behaviour after completing the program.

Sources of objectives

- The general aims of education
- The aims of mathematics education
- Suggestions of experts/ personal experiences
- Nature of the learner and that of the subject
- The societal needs
- Learning and instructional theories
- Already developed list of objectives

Criteria of instructional objectives

- in an attainable form.
- Specific
- clearly stated, not vague or indefinite
- stated in terms of the student
- in terms of observable student behaviour
- Objectives stated should be consistent
- justifiable as significant

Objective based instruction

- goal-oriented activity-meaningful.
- The teaching-learning process is always oriented towards some pre-determined objectives (instructional objectives).
- These objectives make the instruction meaningful.
- help the teacher to select the appropriate learning activities and evaluation strategies.
- A teacher who has clearly defined the purposes of teaching a specific content will be sure about the destination and the expected outcomes of learning. Such an instruction is called objective based instruction.

Interrelationship

- Instructional objectives determine the type of learning activities.
- The effectiveness of the learning experiences to realise the objectives- evaluated through the appropriate devices and these evaluation procedures give evidences about the extent of realization of the objectives.
- Devices for evaluation are determined by the instructional objectives and the learning experiences provided

Objectives of teaching Mathematics with respect to NCF

- Learn to enjoy mathematics.
- Pose and solve meaningful problems
- Use abstraction to perceive relationships, to see structure, to reason out things and to argue the truth or falsity of statement.
- Understand the basic structure of mathematics which offers a methodology for abstraction, structuration and generalisation.

Objectives of teaching Mathematics with respect to KCF

- Mathematization of thought process(the ability to think logically, formulate and handle abstractions)
- Clarity of thought
- Pursue assumptions to their logical conclusions
- Handle abstraction
- Ability and the attitude to formulate and solve problems.

Bloom's Taxonomy of Educational Objectives

- Benjamin S. Bloom and his associates classified the educational objectives into three modes or ways in which people function

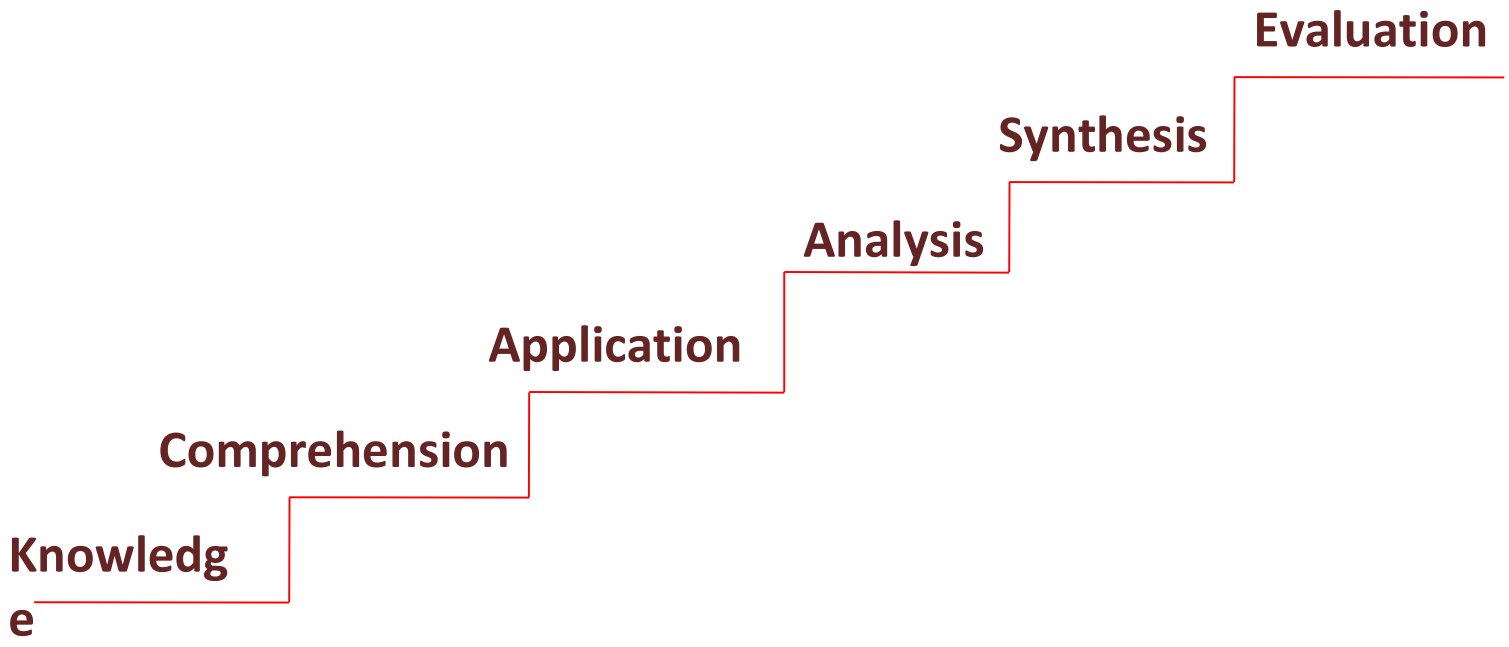
i. Cognitive domain (knowing)

ii. Affective domain (feeling)

iii. Psychomotor domain (doing)

Cognitive domain (1956)

- The intellectual abilities and skills are included in this part. This domain contains six classes of objectives arranged in a hierarchical order.



Knowledge

- knowledge of specifics, of ways and means of dealing with specifics and of universal and abstraction.
- Knowledge acts as the basis of all other objectives.
- Higher level objectives can be attained only if the knowledge level is acquired.

Comprehension

- Understanding/ comprehension means the ability to organize and arrange materials mentally.
- understanding of the terms, facts, principles, generalisation, translation, interpolation, extrapolation and interpretation.

Application & Analysis

- higher order ability to apply the already learnt materials in new, unfamiliar situations.
- Analysis means analysis of elements, relationship and of organisational principles.
- It is the ability to break up a given communication in to its elements in order to see the relationship between the ideas more explicitly.

Synthesis & Evaluation

- Synthesis involves synthesising /formation of a unique communication/ plan/ relationship.
- Evaluation- the highest level- judgment in terms of internal/ external criteria.

- Realisation of these objectives verified through certain behaviours of the learner which are observable and measurable - SPECIFICATION

Affective domain

- Affective domain deals with objectives concerned with the development of interest, attitudes, opinions, appreciation, values and other emotional sets. Krathwohl et al (1964) classified the objectives under this domain as
- ***Receiving, Responding, Valuing, Organisation and Characterisation***

Psychomotor domain

- This domain contains objectives concerned with the development of motor/ manual skills. Based on the concept of co-ordination between different acts, Dave (1970) classified psycho-motor domain as
- ***Imitation, Manipulation, Precision, Articulation and Naturalisation***

Revised Bloom's Taxonomy

- RBT by Lorin Anderson, 2001
- Representatives of three groups-cognitive psychologists, curriculum theorists & instructional researchers, and testing and assessment specialists
- two dimensional frame work: Knowledge and cognitive processes.
- A table with this Knowledge and Cognitive process dimensions –Taxonomy table.

RBT & Original Taxonomy

1) **Change in terminology**

- Remember, Understand, Apply, Analyse, Evaluate and Create.
- **Remember: Retrieving, recognizing, and recalling** relevant knowledge from long-term memory.
- **Understand:** Constructing meaning from oral, written, and graphic messages through **interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining.**

- **Apply:** Carrying out or using a procedure through **executing, or implementing.**
- **Analyze:** Breaking material into constituent parts, determining how the parts relate to one another and to an overall structure through **differentiating, organizing, and attributing.**
- **Evaluate:** Making judgments based on criteria and standards through **checking and critiquing.**
- **Create:** Putting elements together to form a coherent or functional whole; reorganizing elements into a new pattern or structure through **generating, planning, or producing.**

2) **Change in Structure**

- All the original subcategories replaced with the corresponding gerunds, the ‘Cognitive processes’
- **Remember-**Recognising & Recalling
- **Understand-** Interpreting, Exemplifying, summarising, Inferring, Classifying, Comparing & Explaining.
- **Apply-** Implementing, Executing
- **Analyze-** Differentiating, Organising, Attributing
- **Evaluate-** Checking, Critiquing
- **Create**-Generating, Planning**,** Producing

3) **Change in Emphasis**

- The revision of Bloom's taxonomy emphasises explanation and description of subcategories.
- The uni dimensional approach changed to two dimensional
- The linear pattern of expressing objective changed to tabular form.
- Bloom's Taxonomy – class room teaching'
- RBT- in a broader perspective; curriculum planning, instructional and assessment.
- Provides explanation and description of subcategories.

Knowledge dimension

- **Factual Knowledge**:- Knowledge of terminology and Knowledge of specific details and elements
- **Conceptual Knowledge**: Knowledge of classification and categories, Knowledge of principles and generalisations and Knowledge of theories, models and structures.
- **Procedural Knowledge**: Knowledge of subject specific skills and algorithms, Knowledge of subject specific techniques and methods & Knowledge of criteria for determining when to use appropriate procedures.
- **Meta cognitive knowledge**: Strategic knowledge, Knowledge about cognitive tasks, including appropriate contextual and conditional knowledge & Self knowledge

The Cognitive Process Dimension

- The six categories emphasised in the original taxonomy than the sub categories, but in RBT, importance to the 19 specific cognitive processes under these six categories.
- the six categories are hierarchical in nature based on its complexity, but not as strict as in the original in order to allow the categories to overlap one another. (In Understand, the cognitive process ‘explaining’ may be more complex than the ‘executing’ in apply).

2 Mark Questions

1. Write any two objectives of teaching mathematics with respect to KCF.
2. Write any two specifications of the objective Understanding.
3. Write any two values of learning mathematics at secondary level.
4. What is meant by instructional objectives and specification and give one example for each?

4 Mark Short Essay

1. What are the objectives of teaching Mathematics at senior secondary level?
2. How will you impart Mathematical values through teaching.
3. Write the major differences between Bloom's Taxonomy and its revised form.
4. Briefly explain the values of teaching mathematics.
5. Explain taxonomy of instructional objectives in Affective domain.

10 Mark Essay

1. What are the disciplinary and cultural values of learning Mathematics and examine whether these values are being realized as a result of the instruction being imparted in our schools.

B Ed. I. Sem. EDU.05.10 THEORETICAL BASES OF TEACHING MATHEMATICS

UNIT 3
MICRO TEACHING

Group Members

1) Sahla T P
2) Fathima Azeez C P M
3) Dilfa P

TEACHING:MEANING AND DEFINITION

MEANING

Teaching is a process in which learner , teacher curriculum and other variables are organized in a systematic way to attain some pre determined goals .

DEFENITION

“Teaching is a form of interpersonal influence aimed at changing the behavior potential of a another person “(N.L GAGE-1962).

PRINCIPLES OF TEACHING

1. **The Principle of Aim.**

   There should be a definite aim for every lesson.
2. **The principle of Activity or Learning by Doing**. The child is active by nature. He has certain urges which impels him to action. He learns by doing.
3. **Principle of Linking with Actual Life and Other Subjects**.

   Learning should be linked with life and other subjects as far as it is convenient to do so.

4. **The Principle of Planning**.

   Good teaching is always well-planned.

5. **The Principle of Interest or Motivation**.

   The principle of interest or motivation is the most important of all principles.

6. **The Principle of Sympathy and Kindly Atmosphere.**

   Good teaching cannot take place in an atmosphere which lacks kindness and sympathy.

7. **The Principle of Creativity**.

   The idea of a good teacher is to make the pupil creative learner.

8. **Principle of Flexibility and Co-operation**.

   Rigid planning is harmful for good teaching. The plan of a lesson must provide scope to make necessary changes, if need be. Teaching should be flexible to meet the unexpected situations, if any, in the classroom.

9. **The Principle of Diagnostic and Remedial Teaching**. Good teaching is diagnostic and remedial. Once the pupil's difficulties have been diagnosed, they should not be left as they are.

10. **The Principle of Looking Ahead**.

    Good teaching looks ahead while it also takes into account the past experiences of the children. An open-minded teacher is always forward looking.

11. **Principle of model presentation.**

The presentation of the material should be really a model one in every way.

12. **The Principle of Selection of material**.

The right selection of material will result into proper teaching and hence desired results will be achieved. This benefits both the teacher and the students.

13. **The Principle of Gradation or division**.

Easy and simple things should come first and difficult and complicated things will occur afterwards.

14. **Principle of individual differences**.

    A good teacher deals with the students according to their individual differences.

15. **The Principle of democracy**.

    The teacher should adopt democratic attitude with students. He should not be dictatorial.

FUNCTIONS OF TEACHING

1. Diagnostic functions.
2. Prescriptive functions.
3. Evaluative functions.

DIAGONOSTIC FUNCTION

- Independent variables remain more active.

PRESCRIPTIVE FUNCTION

- Judgmental process between the teacher & students to decide the teaching strategies & tactics.
- Bring favorable changes in one's behavior , emphasis on feed-back.

EVALUATION FUNCTION

- Focuses on realization of the objectives.
- Measures the effectiveness of prescriptive functions.

MAXIMS OF TEACHING

Teaching activities and instructional procedure are performed by using maxims of teaching. The term 'Maxim of teaching' may be defined as rules for presenting terms and concepts The term 'Maxim of teaching' may be defined as rules for presenting terms and concepts teaching. The maxims of teaching are very helpful in obtaining the active involvement and participation of the learners in the teaching learning process. They quicken the interest of the learners and motivate them to learn. They make the students attentive to the teaching learning process. A good teacher should be quite familiar with them. They are:

- **1. Proceed from the known to the unknown:** The most natural and simple way of teaching a lesson if to proceed from something that the students already know to those facts which they do not know. What is already known to the students is of great those facts which they do not know. What is already known to the students is of great putting questions on the subject matter already known to the pupils. The teacher is to proceed step by step to connect the new matter to the old one. New knowledge cannot be grasped in a vacuum.
- **2. Proceed from Simple to Complex:** The simple task or topic must be taught first and the complex one can follow later on. The word simple and complex are to be seen from the point of view of the child and not that of an adult. We would be curbing the from the point of view of the child and not that of an adult. We would be curbing the simpler ones are presented.

- **3. Proceed from Easy to Difficult:** We must graduate our lessons in order of ease of understanding the, Students' standard must be kept in view. This will help in sustaining the interest of the students. In determining what is easy and what is difficult we have to take into account the psychological make-up of the child. Logically viewed one skill may be easy but psychologically it may be difficult. There are many things which look easy to us but are in fact difficult for children. The interest of the child has also to be taken into account.
- **4. Proceed from the Concrete to Abstract:** A child's' imagination is greatly aided by a concrete material. "Things first and words after" is the common saying. Rousseau said, "Things, Things. Things." Children in the beginning cannot think in abstractions. Small children learn first form thing which they can see and handle. Care must be exercised to ensure that the students do not remain at the 'concrete stage' all the time. This is the initial step for children with a view to reach the higher stage of 'abstraction' as they advance in age.

- 5. **Proceed from Particular to General:** Before giving principles and rules, particular examples should be presented. As a matter of fact a study of particular facts should lead the children themselves to frame general rules.
- 6. **Proceed from Indefinite to Definite:** Ideas of children in the initial stages are indefinite and very vague. These ideas are to be made definite, clear, precise and systematic. Effective teaching necessitates that every word and idea presented should stand out clearly in the child's mind a picture. For challenging ideas, adequate use must be made of actual objects, diagrams and pictures. Every possible effort should be made to make the children interested in the lesson.

- 7. **Proceed from Empirical to Rational:** Observation and experience are the basis of empirical knowledge. Rational knowledge implies a bit of abstraction and argumentative approach. The general feeling is that the child first of all experiences knowledge in his day to day life and after that he feels the rational basis.
- 8. **Proceed from Psychological to Logical:** Logical approach is concerned with the arrangement of the subject matter. Psychological approach looks at the child's interest, needs, mental make up and reactions. When we treat a subject logically, we are usually thinking of it from our own point of view and not from the point of view of the child. In psychological approach, we proceed from the concrete to the abstract from the simple to the complex and from, known to unknown.

- 9. **Proceed from Whole to Part:** Whole is more meaningful to the child than the parents of the whole. J.P.Guildford, E.B.Newman and May Seagoe conclude after their research that the ‘whole’ approach is generally better than ‘part’ learning because the material to be leant ‘makes sense’ and its part can be seen by the learner as interrelated. The learner sees a relationship between the central idea of the material to be learned. The ‘whole’ unit or passage for slow learners should be smaller than the ‘whole’ for the fast learners.

- 10. **From Near to Far:** A child learns well in the surrounding in which he resides. So he should be first acquainted with his immediate environment. Gradually he may be taught about thing which are away from the local geography and then take up these, district, state, the country and the world gradually.

- 11. **From Analysis to Synthesis:** Analysis means breaking a problem into convenient parts and synthesis means grouping of these separated parts into one complete whole. A complex problem can be made simple and easy by dividing it into units.

- 12. **From Actual to Representative:** When actual objectives are shown to children, they learn easily and retain them in their minds for a long time. This is especially suitable for younger children. Representative objects in the form of pictures, model etc. should

- 13. **Proceed Inductively:** This maxim almost all the maxims stated above. In the inductive approach, we start from particular examples and establish general rules through the active participation of the learners. In the deductive approach, we assume a definition, a general rule or formula and apply it to particular examples. It must be accepted that in the ultimate analysis maxims are meant to be our servants and not masters. Moreover, by and large, they are interrelated. Different maxims suit different situations. It is, therefore, essential that a judicious use should be made of each maxim. They are means and not end.

Origin and Development of Micro-teaching

- The idea of micro-teaching was originated for the first time at Stanford University in USA by Dwight.W.Allen in 1960's.

Meaning and Definition of Micro-Teaching

- **Meaning**

  Micro teaching is a procedure in which a student teacher practices teaching with a reduced number of pupils in a reduced period of time with emphasis on a narrow and specific teaching skill.

- **Definition**

  "Microteaching is a scaled down teaching encounter in class size and time
  - D.W.Allen(1961)

- “Microteaching is defined as a system of controlled practice that makes it possible to concentrate on specified teaching behavior and to practice teaching under controlled conditions.”
  - D.W. Allen & A.W.Eve (1968)

Objectives of Micro-teaching

- To enable teacher trainees to learn and assimilate new teaching skills under controlled conditions.
- To enable teacher trainees to master a number of teaching skills.
- To enable teacher trainees to gain confidence in teaching.

Characteristics of Micro-teaching

- Microteaching is a highly individualized training device
- It is a student teaching skill training technique and not a teaching technique or method
- Microteaching is micro in the sense that it scale down the complexities of real teaching
  - Practicing one skill at a time
  - Reducing the class size to 5 – 10 pupil
  - Reducing the duration of lesson to 5 – 10 minutes
  - Limiting the content to a single concept

- Immediate feedback helps in improving, fixing and motivating learning
- The students are providing immediate feedback in terms of peer group feedback, tape recorded/CCTV
- Microteaching advocates the choice and practice of one skill at a time

Phases of Micro Teaching

Three phases

1. Knowledge Acquisition Phase (Pre-active Phase)
2. Skill Acquisition Phase(Interactive Phase)
3. Transfer Phase(Post Active Phase)

1. **Knowledge Acquisition Phase(Pre-active Phase)**

- Here the Student Teacher attempts to acquire knowledge about the skill.

- What and why of the skill is learned.

2. **Skill Acquisition Phase(Inter-active phase)**

- On the basis of the knowledge gained trainee prepares a Micro lesson
- Practice the skill
- The presentation is evaluated and gives feedback
- Re-plan and re-teach the skill as per the feedback until get mastery over the skill.

3.Transfer phase (Post Active Phase)

- After acquiring mastery over the skill, the trainees are given opportunity to use the skill in normal class room teaching.

- Here the student teacher integrates different skills.

Steps of Micro-teaching

The Micro-teaching programme involves the following steps:

- ***Step I-*** Particular skill to be practiced is explained to the teacher trainees in terms of the purpose and components of the skill with suitable examples.
- ***Step II-*** The teacher trainee plans a short lesson plan on the basis of the demonstrated skill for his/her practice.
- ***Step III-*** The teacher trainee teaches the lesson to a small group of pupils. His lesson is supervised by the supervisor and peers.

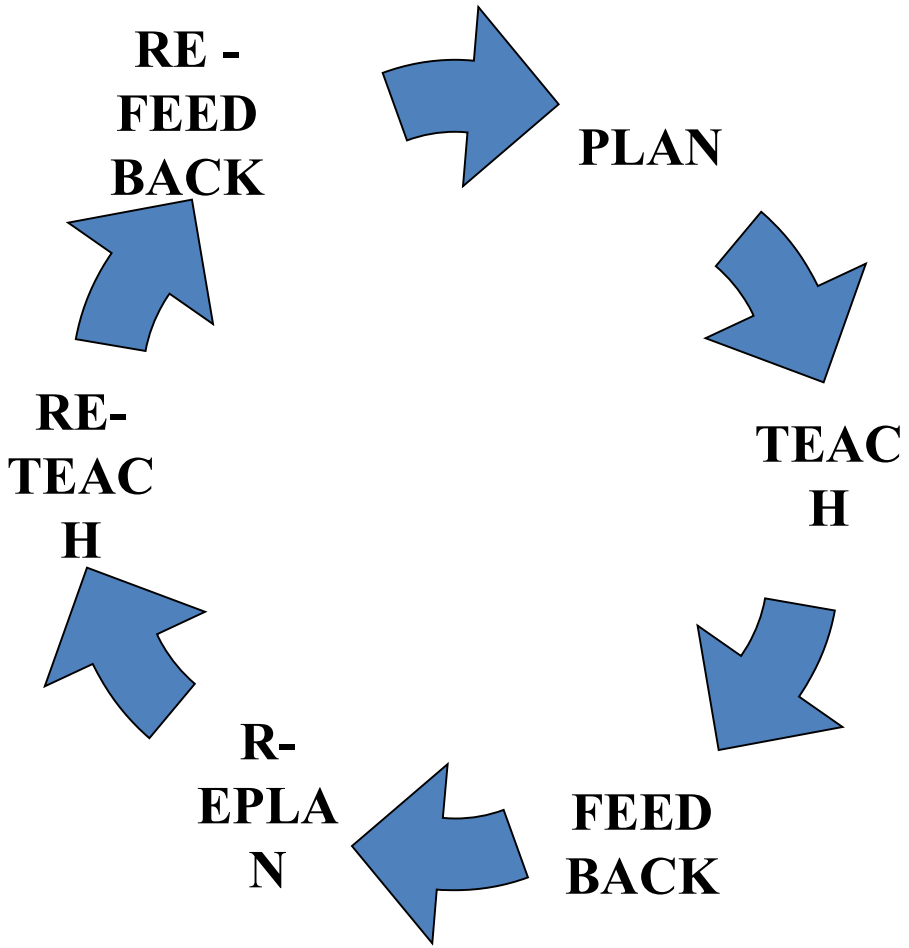
- ***Step IV-*** On the basis of the observation of a lesson, the supervisor gives feedback to the teacher trainee. The supervisor reinforces the instances of effective use of the skill and draws attention of the teacher trainee to the points where he could not do well.

- ***Step V-*** In the light of the feed-back given by the supervisor, the teacher trainee re-plans the lesson plan in order to use the skill in more effective manner in the second trial.

- ***Step VI*** -The revised lesson is taught again according to re-plan.

- ***Step VIII*** -The 'teach – re-teach' cycle may be repeated several times till adequate mastery level is achieved
- ***Step VII***-The supervisor observes the re-teach lesson and gives re-feed back to the teacher trainee with convincing arguments and reasons.

Diagrammatic representation of a Micro-teaching Cycle



Merits of Micro-teaching

- It helps to develop and master important teaching skills.
- It helps to accomplish specific teacher competencies.
- It caters the need of individual differences in the teacher training.
- It is more effective in modifying teacher behaviour.

- It is an individualized training technique.
- It employs real teaching situation for developing skills.
- It reduces the complexity of teaching process as it is a scaled down teaching.
- It helps to get deeper knowledge regarding the art of teaching.

Limitations of Micro-teaching

- It is skill oriented; not Content emphasized.
- A large number of trainees cannot be given the opportunity for re-teaching and re-planning.
- It is very time consuming technique.
- It requires special classroom setting.

- It covers only a few specific skills.
- It deviates from normal classroom teaching.
- It may raise administrative problem while arranging micro lessons.

TEACHING SKILLS

Teaching is a complex skill. It is the integration of many component skills. This component skills can be attained through practice. Even though there are many skills needed for attaining the objectives of teaching.

1. skill of introducing a lesson
2. skill of reinforcement
3. skill of stimulus variation
4. skill of explaining
5. skill of illustrating with examples
6. skill of probing questions
7. skill of using black board
8. skill of questioning

1.Skill of Introducing

Components:-

- Use of previous knowledge:-

  The new learning must be related to the previous learning/experience of the learner. It is the skill of the learner to use the relevant previous knowledge of the leaner.

- Use of appropriate device of technique:-

  A teacher has to use appropriate device of technique to introduce a lesson. The various device/technique include questioning, narration, story telling, teaching aids, experimentation, roll playing, visit to place, examples etc.

- Maintenance of logical continuity:-

  Teacher should ensure the logical continuity of the ideas while introducing a lesson
- Relevance of verbal/non verbal behaviour:-

  Whatever teachers do in the class must be relevant and it should contribute maximum to the introduction of the topic. Any behaviour of the teacher should be for

  1. To test the previous knowledge of the learner.
  2. To integrate previous knowledge with the new one
  3. To bring out the significance of the new lesson.
  4. To catch attention or to establish cognitive and affective interaction with pupils.

2.Skill of Reinforcement

It is a psychological concept. There are two types of reinforcement.

1. Positive reinforcement
2. Negative reinforcement

Positive reinforcement helps the student to strengthen their abilities and motivate them while the other weaken or eliminate good habits.

Components:-

Desirable behaviors:-

- Use of verbal reinforcement:-

  Use of praise words such as good, very good, well done, excellent, fantastic, marvelous, amazing, wonderful, outstanding, repeating the students responses or telling the students to say aloud again.

- Use of +ve non-verbal reinforcement:-

  It refers to all non-verbal teacher behaviours, use of non verbal actions conveying pleasant feeling or approval of pupil responses like moving of head, smiling, clapping, keeping eyes, turning ears or moving towards the student, tapping on the shoulder, writing the responses of the student on the board.

- Use of extra verbal reinforcement:-

  It lies mid way between the verbal and non verbal reinforcement. (Eg:hmmm, wow, aaah..)

Undesirable behaviours:-

- Use of discouraging words:

  Eg:No, incorrect, nonsense, stupid, foolishness etc..

- Use of -ve non verbal reinforcement:-

  It is the expression of dislike through actions and gestures.

  Eg:Frowning, raising the eye brows, moving head with -ve sense, tapping foot impatiently, disapproving stars

- Inappropriate or wrong use of reinforcement:-

Only proper and right use of reinforcement brings encourages results. The following types of re-inforcement should be avoided.

1- Using reinforces when not needed.
2- Not using reinforces when needed.
3- Using less or excess amount than desired.
4- Encouraging only a few responding pupils.

3.Skill of Stimulus Variation:

Components:-

- Teacher movements:

  The teacher move from one place to another during the course of lesson like moving to black board, to pupil, to clarify their doubts etc. The purposeful movement of the teacher capture the attention of the learners. The aimless habitual movements like wandering in the class, up and down movements must be avoided which can be monotonous and some times distractive.

- Teacher gestures:

  Gestures are the movements of the parts of the body to direct attention, to express emotions, to emphasis importance or to indicate shape, size and movements etc.

- Change in speech pattern:

  Sometimes to show emotions or to put emphasis on a particular point, sudden changes in tone, volume or speed of the verbal presentation are brought about. The change in the speech pattern makes the pupil attentive and the lesson become more interesting.

- Change in interaction style:

  When two or more individuals communicate with each other, then interaction takes place. Three styles of interaction are pssible in a class room.

  1. Teacher-class interaction.
  2. Teacher-pupil interaction.
  3. Pupil-pupil interaction.

- Change in sensory focus:

  It refers to the change in sensory channels which pupil uses. Includes changes of sensory focus from listening to looking, from reading to counting, from speaking to doing, from teacher talking to black board writing, etc

There are three types of oral visual switching

Oral----visual

Oral--- oral visual

Visual---oral visual

And vice versa.

- Pausing:

  Teacher can pause for giving emphasis for a concept or the teacher can pause after asking a question so that pupil think of a thoughtful answer. Pausing means deliberate use of silence during the talk.

- Focusing:

  Focusing refers to directing pupils attention to a particular point in which they are required to observe. This can be verbal, gestural or verbal come gestural.

- Pupil movement:

  It represents variation in physical involvement of the student. Here the students are expected to workout on black board, to involve in group activities and construction demonstration and role playing, for holding chart, models etc.

4.Skill of Explaining:

Components:

a. Use of introducing statements
b. Use of explaining links
c. Use of visual techniques
d. Interesting to pupils
e. Covering essential points
f. Use of concluding statements
g. Questions to test pupil's understanding.

5.Skill of Illustrating with Examples:-

Components:

a. Formulating simple examples
b. Formulating relevant examples
c. Formulating interesting examples.
d. Use of appropriate media.
e. Use of inductive/deductive/inductive-deductive approach.

6.Skill of Probing Questions:-

Components:

1. Prompting

   When the pupil gives no response or incorrect responses, the teacher gives hints or clues to read the students to the right answer

2. Seeking further information

   When the pupil response is partially correct or incomplete the teacher helps the pupil to elaborate or clarity or explain the response by asking additional questions.

3. Refocusing

   When the students give the correct response the teacher can relate their responses with something already learned.

4. Redirecting

   Directing the same questions to other pupils when there is a wrong response, incomplete response, partially right response or while seeking further information and so on. This is done to get participation of more students too.

5. Increasing critical awareness

   Using ‘how and why’ type question to increase the critical awareness of the students. Through this the students can understand the logic behind their answer in a better way.

7.Skill of using Black board:-

Components:

a. Legibility
   - Proper spacing between letters and words.
   - Slant of the letter.
   - Capital and small letters of the same size.
   - Size of the letters to be large enough to read.

a. Neatness
   - Letters and words should be in a straight line and parallel to the base of the board.
   - Line spacing should be equal.
   - Avoid over writing.
   - Black board should not look over crowded.

C. Organization of black board

- Systematic planning of space for black board work.
- Adjustment of space for presenting related items in totality (letters and figures).
- Spacing to exhibit the sequence of the items being presented.

d. Appropriateness of black board work

- Continuity in the content matter.
- Brevity in points.
- Proper use of colour chalks to focus ideas.
- Underlining only the important points

e. Precautions for black board work

- Pre-decide things to be written on the black board.
- Ready with the materials acquired-chalk, colour chalk, duster etc.
- Black board should be rubbed before and after using.
- Place the board in appropriate place.
- Don’t cover the board when writing.
- Read the things when writing on black board.
- Use a pointer to focus points.
- Pay attention towards the students while writing.

8.Skill of Questioning:-

Components:

a. Structure

It is the technique of formulation of questions. The following factors should be considered while framing questions

1. Grammatically correct.
2. Relevancy.
3. Specific.
4. Concise
5. Clarity

b. Variety

The question should refer to the different levels of the students. Low and higher order questions should be asked.

C. Pause

d. Voice

e. Speed

2 Mark Questions:

1)Define teaching?

2)what is stimulus variation?

3)what is micro teaching?

4 Mark Questions:

1)Explain maxims of teaching?

2)Explain the types of questioning with examples?

3)what are the different phases of teaching mathematics?

4)Define micro teaching? Describe any three micro teaching skills

EDU.05.10.THEORETICAL BASES OF TEACHING MATHEMATICS

Unit 4

Approaches, methods and techniques of teaching mathematics

Behaviourist Approach

- Behaviourism is the view that behaviour should be explained by observable experiences, not by mental processes
- The teacher decides the sequence of activities or experiences to be received by the students
- The learner is not creative, but only a receiver of knowledge, the main source of which is the teacher

- The class rooms are teacher dominated, repetition oriented and mechanical
- The teacher determines the behavioural changes to be brought among children, plans the activities, executes the activities and then evaluates the learner’s performance through observable behaviour

Constructivist Approach

- The basic idea of constructivism is that the learner must construct knowledge; the teacher cannot supply it
- Learning is active, constructive, cumulative, goal directed, diagnostic and reflexive in which the learner is building an internal illustration of knowledge or personal interpretation of experiences
- **Zone of Proximal Development(ZPD):** The range of tasks that a student cannot do alone but can accomplish when assisted by teacher

- **Scaffolding:**
  The assistance given to the student which allows him to complete the tasks. that they are not able to complete independently
- In classrooms, teacher provide scaffolding by breaking content into manageable pieces, modeling skills, providing practice and examples with prompts and leading the student do the learning activities
- Some processes for effective scaffolding:
- Gain and maintain the learner's interest in the task
- Make the task simple
- Emphasize certain aspects that will help with the solution
- Control the child's level of frustration
- Demonstrate the task

Basic assumptions of constructivism

- Knowledge is constructed from experience
- Learning is personal interpretation of the world
- Learning is an active process in which meaning is developed on the basis of experiences
- Conceptual growth comes from negotiation of meaning, the sharing of multiple perspectives and the changing of our internal representations through collaborative learning
- Learning should be conducted in realistic settings
- Testing should be integrated with the task and not as a separate activity

Characteristics Of Constructivist Learning Environment

- Provides multiple representations of reality
- Multiple representations avoid over simplification and represent the complexity of the real world
- Encourages knowledge construction rather than knowledge reproduction
- Emphasizes tasks in a meaningful context rather than abstract instruction out of text
- It provides environment such as real world settings or case based learning instead of predetermined sequence of instruction
- It encourages thoughtful reflection on experiences
- It enables context content dependent knowledge construction
- Supports collaborative construction of knowledge through social negotiation

Characteristics of constructivist classroom

- Student initiation and autonomy are accepted and encouraged
- Higher level thinking is encouraged
- Students are engaged in dialogue with the teacher and with each other
- Students are engaged in experiences that challenge the hypotheses and encourage discussion
- Teacher asks open ended questions and allows time for responses
- The class uses raw data, primary sources, and manipulative, physical and interactive materials

Educational implications

- Learners build new knowledge upon the foundation of previous learning
- Learners are not passive; they remain active in the process of applying current understanding. Noting down relevant elements in new learning experiences judging the consistency of prior and new knowledge based on the judgment, modifying the knowledge
- Constructivist teacher is never a ‘sage on stage’ but he takes the role of ‘guide on the side’ who provides students with opportunities to test the adequacy of their current knowledge

Problem Based Learning (PBL)

- It is an exciting alternative to traditional class room learning
- With PBL, your teacher presents you with a problem, not learners or assignments or exercises
- Since you are not handed ‘content’, your learning becomes active in the sense that you discover and work with content that you determine to be necessary to solve the problem
- Teacher acts as facilitator and mentors, rather than the source of “solutions”

Problem based learning will provide you with opportunities:

- Examine and try out what you know
- Discover what you need to learn
- Develop your people skills for achieving higher performance in team
- Improve your communication skills
- State and defined positions with evidence and sound argument
- Become more flexible in processing information and meeting obligation
- Practice skills that you will need after your education

Steps of PBL

1. Explore the issues
2. List “what do we know?”
3. Develop, and write out, the problem statements in your own words.
4. List out possible solution
5. List actions to be taken with a timeline
6. List “what we need to know”
7. Write up give solutions with its supporting documentation, and submit it
8. Review your performance
9. Celebrate your work

Heuristic Method

- The term ‘heuristic’ is originated from the Greek word ‘heurisco’ which means ‘I find’
- Prof. H. E Armstrong originated this method of teaching science
- Here, Child is in the role of discoverer and not the receiver of knowledge
- The learner selects his own path and proceeds according to his own reasoning

- Here teacher is not in the role of problem solver, but he enables the child to solve the problems himself
- Learners may be ignorant of some important ideas or many of these ideas will be new to them. So teacher should interfere in the process of learning by helping them to be in the correct path. Teacher has to ask true heuristic questions in order to help them arrive at the correct answers

Principles underlying the heuristic method

- Principle of activity
- Principle of logical thinking
- Principle of proceeding from known to unknown
- Principle of purposeful experience
- Principle of self-thinking and self-study

Merits

- It is psychologically sound
- It focuses on the active, original and constructive tendencies of the child
- It develops a sense of confidence among children
- Develops attitude of scientific thinking
- Promotes self discipline among learners
- Leaves little scope for forgetting
- The learner is active in the process of learning, he becomes an active discoverer of information
- Teacher pupil relationship is strengthened
- Individual can learn at his own pace
- Learner gets clear understanding of what he has learned

Demerits

- Not suitable for below average students who are poor in independent thinking
- Very talented teachers are needed for guiding the students
- Through preparation is needed on the part of the teachers
- Too much time will be taken for investigation and so it is a slow method
- Individual checking is needed to ensure that each students has arrived at the expected result
- Not appropriate for large classes
- Not suitable for all topics
- If not properly guided, the learner may become disgusted and loss his confidence and become more dependent
- The true spirit of the method will be lost if the student copy the answer from books or other materials

Methods of Teaching Mathematics

Inductive method

- It is the method of inductive reasoning-proving universal truths by showing that if it is true for particular case and is further true for a reasonable adequate number of cases, it is true for all such cases. This process is known as induction
- We proceed from particular to general and concrete to abstract

Example:

Ask the students to measure the interior angles of different triangles and add them. Do the same for any triangle, then they will arrive at the generalization that ‘ sum of the interior angles of a triangle is equal to two right angles’.

Merits

- It is psychological method in which one starts from particular cases and ends at generalization
- It facilitates understanding as it remains no doubts in the minds of the learners
- As it is logical, develops critical thinking
- Stimulates learning by self as it encouraged active participation of students
- It is natural method of making discoveries
- Avoid the chance for cramming and rote learning
- It is based on actual observation, thinking and experimentation
- It increases teacher - pupil interaction

Demerits

- Not applicable to all mathematical generalization. It is applicable only in those cases where the actual observation of the particular cases is feasible
- As a method, it is incomplete by itself. Only limited number of cases are used for arriving at generalization, which may or may not be true for all the cases
- It is time consuming and laborious
- The scope of application of formula in new situation is limited in this method
- Need not be successful at higher levels
- All students may not be able to follow the generalization process

Deductive Method

- One starts with a formula or rule or generalization and applies to specific cases
- Proceeds from general to specific, abstract to concrete, formula to examples
- Students are asked to accept a generalized truth or an established formula and then apply it in specific situations to solve problems

Merits

- Short and time saving
- Even below average students can follow this method
- It is suitable for almost all topics
- Provides sufficient practice in the application of mathematical rules and formulae
- Speed and efficiency in solving problem can be enhanced
- Very appropriate while doing practice and revision
- It is applicable for lower as well as higher levels of learning

Demerits

- It is unpsychological to start with an abstract idea
- Role learning is encouraged
- Students may not be active learners
- More emphasize is given to memory
- Does not encourage logical thinking and discovery
- When the pupil forgets a formula he is enable to rediscover it

Analytic Method

- The word analytic is derived from the word analysis which means breaking up or separate things that are together
- Analysis starts with what we have to find out and traces the connection between it and data
- Proceed from unknown to known
- From conclusion to hypothesis
- We start from what's to be determined or what is to be provided
- It is called method of discovery

Merits

- It is psychological
- It is logical and hence leaves no doubts
- Develops reasoning power and avoid rote learning
- Pupil participation is encouraged
- Develops the spirit of enquiry among learners
- Self confidence of learner can ve developed
- Helps the development of intellectual power
- It is helpful in the development of scientific attitude and originality among learners

Demerits

- It is lengthy and time consuming
- It does not develop speed and efficiency in solving problem
- Some problems cannot be solved with this method
- Memory is not promoted
- Does not present the fact neatly and systematically
- Below average students may feel difficulty in analyzing the problem

Synthetic Method

- This method is opposite to analytic method
- The term synthetic is derived from the word synthesis which means combine together
- Child proceeds from known to unknown
- Facts already known are applied to new situations so that combination of known facts helps up to find new facts

Merits

- Short, concise and elegant
- It glorifies memory as the student has to memorize the different procedures to be followed in different situation
- It is economical with reference to time
- It proceeds from known to unknown

Demerits

- Many doubts are left in the learner's mind
- Rote learning encouraged
- More emphasize is given to memory, not for the reasoning or thinking
- Rare chance of complete understanding of the topic
- Makes the learner over burdened
- If the learner forgets a single step, not easy to recollect it.

Project Method

- Developed by Kilpatrick
- It is based on pragmatic philosophy works on the assumption that different branches of knowledge are different aspects of one whole and this knowledge became meaningful if they have some application

Definitions

- A project is a whole hearted purposeful activity proceeding in natural (social) environment.
  -Dr. Kilpatrick
- project is a problematic act carried to completion on it's natural setting **-Stevensen**
- Project is a bit of real life that has been imparted in to the school **-Ballard**
- It is a unit of educative work in which the most prominent features is some form of positive and concrete achievements. **-Sneddon**

Types of project

- **Producer type-** emphasis is on the construction of a materials, objects or article
- **Consumer type-** importance is on receiving direct/vicarious experiences
- **Problem type-** to solve problem through intellectual participation, enquiry and reflection by the learners
- **Drill type-** focus on mastery of a skill

Characteristics of good project

- Should be closely related to the curriculum
- Suitable to learner and community
- It should be timely
- The experience obtained through a project should be useful
- Economical- not waste time and money of student
- Create interest and curiosity among students
- It should be challenging to the learner

Principles

- Principle of motivation
- Principle of activity
- Principle of utility
- Principle reality
- Principle of feasibility
- Principle of experience

Steps

1. Providing situation
2. Selecting and purposing
3. Planning
4. Executing
5. Evaluating
6. Recording

Role of teacher

- Giving suitable situation to students to select project
- Giving guidance in the selection and planning
- Developing a democratic environment in which co-operation among students is ensured
- Supervising students while carrying out the project
- Verifying the records maintained by the students
- Be a guide, philosopher and friend instead of a director or commander

Merits

- Independent work and individual development is possible
- Critical thinking possible
- Life oriented, meaningful learning occurs
- Active participation of learner is ensured
- Skill like observation, interpretation act are developed
- Upholds dignity of labour
- Social interaction is promoted
- Self confidence and self discipline developed
- Caters individual differences

Demerits

- Uneconomical with regarding time and energy
- Mathematical thinking and reasoning is not fostered
- Scarcity of resources
- Syllabus cannot be covered
- Teaching disorganized
- Highly creative, resourceful teachers needed
- All are not receiving the same kind of experiences.

Laboratory Method

- Students are asked to do some experiments and practical work and verify some mathematical truth
- It is based on psychological principles of learning like ‘learning by doing’, ‘learning by observation’, from ‘simple to complex’ etc

Merits

- Based on psychological principles
- Makes learning meaningful
- Student participation is ensured
- Develops the attitude of discovery among students
- Fosters self confidence and self reliance among students
- Gives the students chances to deal with concrete materials
- Reduces the abstract nature of mathematics
- Knowledge acquired through this method is long lasting
- Students retain the knowledge for a long time
- Suitable for below average and average students

Demerits

- All topics in mathematics cannot taught through this method
- Mental development of the learner is not much concerned
- It is expensive as there is need of well equipped laboratory
- Demands highly competent teachers
- It is not economical with respect time
- It is not applicable to higher classes
- Not possible to large classes

Problem solving method

- Problem solving is the ultimate purpose of teaching mathematics
- Students learn mathematics in order to solve a great variety of problem together with acquiring knowledge, basic skills and understanding mathematical concepts

Problem

- A question raised for inquiry, consideration or solution … a source of perplexity
- Originated from the Greek word ‘problema’ meaning ‘something thrown forward’

Characteristics of problem

- It should be a question or situation
- It should be accepted by the student
- It should be challenging to students
- It should be real rather than artificial one
- It should be educationally significant
- It should have both practical and social values
- It should be clear and free from ambiguities
- It should provide best mental discipline to the students
- It should be related to other subjects, the subunits, unit and the course

Steps in problem solving

1. Identifying and defining the problem
2. Gathering data in systematic manner by analyzing the problem
3. Formulating tentative solution
4. Arriving at the correct solution
5. Verifying the results

Maxims of teaching problem solving

- Make sure that students understand the problem
- Help students to think rationally for creating a plan
- Assist the students in gathering information by having them analyze the given condition
- Provide them with an atmosphere conductive for solving problem
- Once students have obtained a solution, encourage them to reflect on the problem and the means of arriving at solution
- Encourage students to seek present alternative ways of solving problems
- Challenge students to investigate various forms of the given problem

Merits

- Provides real life experience to students
- Develops in pupil good habits of planning, thinking, reasoning and independent work
- Develops initiative and self responsibility among students
- It takes in to account of individual differences
- Helps the students to develop reflective thinking
- It helps the students to approach future problems with confidence
- It builds a mental attitude for reasoning based on critical thinking

Demerits

- Not all students are problem solvers
- It becomes monotonous if used frequently
- It is time consuming and it is not possible to cover syllabus on time
- Success of this method depends on mathematics teacher who is well versed in critical thinking and reflective thinking, but not all mathematics teachers are well versed in this
- References and resource material may be difficult to get.

Techniques of teaching mathematics

Questioning Technique

"Good learning starts with questions, not answers"

In questioning technique teachers uses questions as a tool to promote inquiry, thinking, and ultimately learning. Questioning is often called the hub of teaching process.

Some functions of good question

1. They encourage thinking on the part of pupils
2. They encourage pupil participation in class activities
3. They can be excellent and motivational devices
4. They encourage pupils to analyse and synthesis ideas

5. They help the teacher to evaluate the effectiveness of this activities
6. They help to guide the direction of the lesson development
7. They may help pupil to make generalizations and formulate new hypothesis

Thumb rules of effective questioning

- A few general rules of thumb for designing effective questions are:

1) **One at a Time:** Have only one question in the question.

2) **Simple to Complex:** Ask questions that progress from simple to complex.

3) **Clear and Concise:** The questions should be clear and Concise

4) **Start with a question word.** (who, when, what, where, why, how)

5) **Ask an actual question.**

6) **Assume the answer. (Ask, "Who can tell me…," not, "Can anyone tell me…"**

7) **Stock Questions:** Ask one sequence of questions in a row. Ask versions of the same question.

8) **Break it down**: Break complex questions to simple, one after the other.

Brainstorming

Brainstorming is the name given to a situation when a group of pupil meets to generate new ideas around a specific area of interest. Brainstorming is a teaching technique which has wider application

The meaning of Brainstorming

In context to teaching, brainstorming is a technique or tool of teaching used by the teacher in which maximum or all the students participate by responding or presenting views on one topic. This technique encourages new ideas among students which would never have happened under normal circumstances.

Brainstorming can be explained in following ways:-

- It is a process designed to obtain the maximum number of ideas relating to a specific area of interest.
- It is a technique where a group of pupil put social inhabitations and rules aside with the aim of generating new ideas and solutions.
- It is a technique that maximizes the ability to generate new ideas.

Brainstorming can either be traditional or advanced.

(a) Traditional brainstorming

- Traditionally for Brainstorming pupil gather in a room and forward their ideas as they occur to them. They are told to lose their inhabitations and no ideas shall be judged. Here pupil should build on ideas called out by other people.

- **(b) Advanced Brainstorming**

It is an extension of the traditional brainstorming and makes the whole process easier and effective. Advance brainstorming uses new process and new techniques to reduce inhabitations, for example, creative and lateral thinking technique.

Brain storming software

New material for simulation and recording ideas.

- **Brainstorming in education**
- In the field of education brainstorming is a large or small group of activities that encourage the student to focus on a topic and contribute to the free flow of ideas. In this process

- Teacher begins the session by posing a question, problem or by introducing a topic.
- The student then expresses possible answers, relevant words, and ideas.
- The contribution is accepted without criticism or judgment and is then summarized on a white board by the teacher.
- These ideas are examined, usually in an open class discussion format.

- **Purpose of Brainstorming**

- To focus student attention on a particular topic.
- To generate particular ideas.
- To teach acceptance and respect for individual differences.
- To encourage the learner to take a risk in sharing their ideas and opinions.
- To demonstrate to the student that their knowledge and abilities are valued and accepted.
- To provide an opportunity for students to share ideas and expand their knowledge by building on each other's

Characteristics of Brainstorming

- It is an intellectual activity.
- Maximum or all students can participate.
- Each student gives their personal view/ideas.
- Each idea is neither right nor wrong.
- It involves divergent thinking.

Brainstorming process

- First, a small group of students is formed. They are asked to sit in a group and are provided with a particular issue or topic.

- Teacher, as the group leader, then ask group members to think about the problem and give their ideas. They are advised to find as many solutions to the problem as they can find. They are instructed not to criticize others ideas but they are free to make attentions to others ideas. Students are encouraged to put forward suggestions without hesitation even if they seem to come up with unusual and unorthodox ideas.

- Students' ideas are to be listened and accepted patiently, without passing any judgment or comment of any sort until the session is over.

Advantages

- It stimulated and provides varied instructional approach.
- Highly motivating.
- Increase task focus.
- Promotes spontaneity and creativity.
- Efficient and procedure.
- Involves participants in ownership of ideas.
- Encourages creativity.

Assignment As A Teaching Technique

The Assignment method is the most common technique of teaching especially in teaching of Science. It is a technique which can be usually used in teaching and learning process. It is an instructional technique comprises the guided information, self learning, writing skills and report preparation among the learners. The Assignment method is an important strategy in teaching and learning process.

OBJECTIVES

- It provides good training for information seeking and retrieval behaviour.
- It inculcates the self learning attitude among the students.
- It provides information analysis and research attitude to the learners

• It develops the learning experiences from various sources.

STEPS / STAGES IN ASSIGNMENT

• The assignment must be lesson concerned and related with the text books and curriculum.

• The topic / unit of the assignment must be explained with the availability of resources.

• The core of the subject or unit must be clarified.

• The hard and difficult portions of the assignment need to be explained well.

• The topics / units irrelevant to the assignments must be defined very well

FEATURES OF GOOD ASSIGNMENT

• Assignment must be relevant to the subject taught to the student.

- This should reflect the affinities with the subject contents in the text book concerned.
- Assignment must be simple and enable the students to complete it within the stipulated time.

• Assignment must avoid ambiguous, complex information and instructional structure.

• Objectives of the assignments must be clear and definite.

Teaching for understanding proofs

Direct Proof

Consider an implication: $p \rightarrow q$

- If p is false, then the implication is always true
- Thus, show that if p is true, then q is true

To perform a direct proof, assume that p is true, and show that q must therefore be true

Indirect Proofs

Consider an implication: $p \rightarrow q$

- It's contrapositive is $\neg q \rightarrow \neg p$
- Is logically equivalent to the original implication!
- If the antecedent ($\neg q$) is false, then the contrapositive is always true
- Thus, show that if $\neg q$ is true, then $\neg p$ is true

- To perform an indirect proof, do a direct proof on the contrapositive

Proof by mathematical induction

- Proof by induction is just like an ordinary proof in which every step must be justified . however it employs a neat trick which allows you to prove a statement about an arbitrary number ‘n’ by first proving it is true n is 1,and then assuming it is true for n=k and showing it is true for n=k+1.

Proof By Contradiction

- Given a statement p, assume it is false
  - Assume ¬p
- Prove that ¬p cannot occur
- A contradiction exists
- Given a statement of the form p → q
- To assume it's false, you only have to consider the case where p is true and q is false

Proof By Causes

- Proof by causes is a method of mathematical proof in which the statement to be proved is split into a finite number of cases and each case is checked to see if the proposition in question holds. here ,one can go over all the possible cases for each member of the unite domain. the final result of this exercise: you prove or disprove the theorem but you could be definitely exhausted. a proof by exhaustion contains two stages:

1. a proof that the cases are exhaustive; i.e., that each instance of the statement to be proved matches the conditions of(at least) one of the cases.
2. A proof of each of the cases.

Proof by Contra positive

- Proof by contrapositive takes advantage of the logical equivalence between “p implies q” and “not q implies not p”. for example , the assertion “ if it is my car , then it is red” is equivalent to “ if that car is not red , then it is not mine “.

So , to prove “ if p ,then q” by the method of contrapositive means to prove “if not q , then not p”.

Disproof By Counter Example

• Given a universally quantified statement, find a single example which it is not
true

• Note that this is DISPROVING a UNIVERSAL statement by a counter example

2 mark questions

1. Give an example for proof of contradiction in mathematics.
2. Write two merits of problem solving method.
3. Mention any two important features of problem based learning

4 mark short essay

1.explain the constructivist approach in the teaching of mathematics.
2.Write the steps in the project method with illustration
3.Explain heuristic approach in mathematics learning
4.Explain mathematical induction with example.
5.Mention the significance of laboratory method for teaching mathematics
6.Briefly explain different steps in problem solving method? Enumerate the merits and demerits of problem solving method
7.Compare the behaviourist and constructivist approach.
8.Discuss the heuristic method of teaching mathematics. illustrate its application with a help of a suitable example.

9.briefly explain different steps in project method with the help of an example . enumerate the merits and demerits of project method.

10.Distingiush between inductive –deductive methods of teaching mathematics

10 mark essay

1.Explain laboratory method of teaching mathematics with suitable example.
2.Illusrate analytic –synthetic methods of teaching mathematics with examples
3.Illusrate inductive –deductive methods of teaching mathematics with suitable example . also mention its merits and limitations.

BEd First sem Edu. 05.10 Theoretical bases of teaching Mathematics.

Unit 5

MATHEMATICS CURRICULUM

Group Members

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Meaning of curriculum

- ‘curriculum’ - Latin word ‘currerre' - ‘run' / ‘path’
- A course to be run for reaching a certain goal /destination
- **Cunningham(1951) -** “curriculum is a tool in the hands of an artist(teacher) to mould his material (pupils) according to his ideals (objectives) in his studio (school)”
- **Pyne- “**curriculum consists of all situations that the school may select and consciously organize for the purpose of developing the personality of its pupils and for making behavioural changes in them”

- **Secondary education commission 1953-“**Curriculum is the sum total of the experiences that a pupil receives through the manifold activities that go in the school in the classroom, library, laboratory, workshop, play ground, and in the numerous informal contacts between teachers and pupils.”

Major features of a curriculum

- Curriculum is pre planned.
- It has four bases - social forces, knowledge of human development, the nature of learning and the nature of knowledge.
- The purpose of the curriculum is reflected in the list of objectives accompanying it.
- Every learner has an actual curriculum.
- Curriculum facilitates planning by teachers

Types of curriculum

1. **Traditional curriculum**

- It is an educational curriculum which follows established guide lines and practices.
- This type of curriculum gives a “quantum of knowledge” rather than “totality of experience”.

2. Activity centered curriculum

- It is based on learning by doing.
- Children acquire knowledge through activities .
- Less role of verbal instruction.

3. Experienced centered curriculum

- The experience centered curriculum lays emphasis on the development of reasoning, inquiry or exploration.
- This curriculum cannot be fixed in advance.

4. Life centered curriculum

- It includes life experiences and life situations as it is the transmission of life interns of its value,standards and modes.
- This type of curriculum focuses on Dewey's view education is preparation for life through life experience.

5. Balanced curriculum

- If all the principles governing the construction of curriculum is kept in view, the result will be a balanced curriculum.
- It takes to consideration all aspects of man's development and human activity

6. Hidden curriculum

- Process that involves transmission of norms and values as well as body of socially approved knowledge.
- People learn things that are not actually taught in the formal curriculum .

7. Undifferentiated curriculum

- There is certain amount of common body of knowledge and experience which is essential for all normal children irrespective of their sex and aptitude.
- A curriculum embodying such knowledge and experience is called undifferentiated curriculum.

8. Differentiated (diversified) curriculum

- In order to meet the varying abilities of children, differentiated curriculum is essential.
- Here the curriculum is adapted to the tastes and talent of learners.

Curriculum Development

Curriculum development involves four major process. There are

1. Construction : Planning the experiences to be utilized
2. Organization : Organising the experiences into a programme
3. Execution : Implementing the program.
4. Evaluation : Evaluating the curriculum.

Princples of Mathematics curriculum construction.

1) **Principle of child centeredness.**

- Curriculum should be based on the present needs and capabilities of the child.
- Curriculum should be according to the needs, interest, ability, aptitude, maturity of the pupils.

2) **Principle of community centeredness.**

- Curriculum Should ensure optimum utilization of community resources, development of ability to adjust with environment.
- Pupil should understand that they are the members of _community.

3) **Principle of activity centeredness**

- Curriculum must be full of activities.
- ‘Learning by doing ‘should be emphasized in the curriculum.
- Child learn more from activities, it should be connected with the child’ desires and needs the societal requirements.

4) **Principle of integration**

- Mathematics curriculum should not be considered as entirely different from that of other subjects.
- It should be in conformity with the curriculum of other subjects.
- There should be continuity and coherence within mathematics and with respect to other subject as well.

5) **Principle of forward looking**

- The curriculum has to enable the child to adjust to his immediate environment.
- Capability of adjustment in different circumstance of life
- Equip child to face the challenges that come in their future life.

6) **Principle of conservation**

- Curriculum should include those activities which help the individual in preserving and transmitting Tradition.
- Preserve the culture and tradition and it should be transmitted to next generation.

7) **Principle of maturity**

- Curriculum should be suited to the mental and physical maturity of the pupil.

8) **Principle of elasticity and variety**

- Curriculum should not be rigid but should be flexible to suit the changing needs of the peopleand the society.
- The curriculum should include variety of activities keeping in view the individual needs of the children.

9) **Principle of balance**

- The curriculum must maintain proper balance among the direct and indirect experiences, individual and social aims.

10) Principle of totality of experience

- Apart from the subject of study, the curriculum should include many other activities to provide variety of experience of students.

11) Principle of utility

- Vocational and technical basis should be maintained in the curricular activities.

12) <u>Principle of scientific thinking</u>

- The curriculum should enable the students to think scientifically, to learn problem solving strategies, to develop critical thinking.

13) <u>Principle of individual differences</u>

- Curriculum should realize the needs of average, intelligent and weaker students.
- The scope of individual difference should be considered.

14) <u>Principle of creative thinking</u>

- The curriculum should develop the constructive power of individual.
- Thebcurriculum has to develop original and rational thinking.

Curriculum Organization

After selecting the content and activities of the curriculum, they are to organized by maintaining mathematical sequence and continuity.

Principle of curriculum organization.

1) **Principle of correlation**

While organizing the mathematics curriculum the correlation of the content with dialy life, other subjects, different branches of mathematics and with the different topics of the same branch should be considered.

2) **Principle of vertical correlation**

The curriculum of one level should be in continuation with that of the lower classes.

3) **Principle of simple to complex**

The activity should be arranged in the order of difficulty. That is, it follows simplex to complex.

4) **Principle of psychological aspects**

The mathematics curriculum should be organized based on the principle of development and learning.

Approaches Of Curriculum Organization

1) **Logical Vs Psychological approach**

- Logical approach focuses on the logical sequence of the content. This will help in maintaining the link and sequence of topics which are useful and meaningful for the child.
- Psychological approache stresses on the principle of child centerdness. Here subject is arranged based on the principle of psychology.

2) **Concentric Vs Spiral**

- In concentric approach, during the initial level, elementary knowledge is given, and in successive years, more and more details are added.
- In the spiral approach, continuity of the topic is considered together with gradation. Elementary concepts are presented at one level, gaps are filled in the next and more and more gaps are filled in the later years.

3) **Topical Vs Unitary approach**

- In topical approach, the subject matter is grouped as a topic which contains

- The materials related to specific area of the subject. The content in a topic will be sequentially arranged so as to get a holistic view of the materials.
- In unitary approach, the topic is divided in to meaningful units based on the unifying principle of the content, and these units may be properly linked and graded to form a spiral curriculum.

Mathematics Curriculum Reforms

1.School Mathematics Study Group (SMSG):-

- The first attempt to reform elementary currirulum was occured in 1959 when the School Mathematics Study Group (SMSG) sponsored a conference in Chicago on elementary school mathematics
- SMSG started in the summer of 1958 in USA.
- Project was funded by the National Science Foundation.
- First session was held at Yale University under the dictatorship of Prof. E. G Begle.

Major aims were:

- To develop a mathematics curriculum for grades 7 through 12 that would consist of accurate and challenging mathematics for the college capable.
- To prepare text books for the course

Attempts to improve elementary school mathematics:

- In 1959 conference, a detailed programme outline for grades 4,5 and 6 were made
- In 1960, at Stanford University, the text books for these grades were prepared.
- Canters to test the textual materials and to conduct in - service programme for the teachers were established.
- Experiments was done at 27 centres.
- Over 12000 students used the material.
- Feedback from the teachers were collected and materials revised for the next 2 years.

- SMSG began the curriculum revision with grades 7 through 12 and then went to 4 through 6 and finally kindergarten through 3.
- Some new topics were introduced with new approach for the new concept.
- More emphasis given to structure of mathematics.
- Mathematics concepts were introduced on informal and intensive level.
- SMSG completed its set of text books in 1966.
- Initiated development of new flexible, sequential programme for grades 7 through 12.

Characteristics of SMSG -secondary school curriculum :

- This curriculum is devoted solely to mathematical concepts.
- All citizens should know in order to function effectively in our society.
- Usual grade placement of mathematical topics is ignored.
- Topics from Arithmetic, Algebra and Geometry are introduced.
- Certain topics such as functions, co-ordinate geometry, rigid motions, computer mathematics, probability and statistics are included.

- A strong attempt to make clear to the students the relevance of mathematics to problems of real world.
- In 1959,SMSG formed a panel of educators and mathematicians to plan programme for students of average and below average ability in mathematics
- In APRIL 1964, the SMSG held a Conference on Mathematics Education for below average achievers to determine what role SMSG should play.
- They recommended establishing a series of institutes to deal specifically with low- ability child.
- They produced a special edition of secondary school mathematics for 7th and 8th grade low achievers presenting same general content in regular programme. But rewritten to appeal to students with low mathematics achievement.

2.School Mathematics Project (SMP):-

- SMP was founded in 1961,under the Chairmanship of Bryan Thwaites with the objective to device radically new mathematics courses, with accompanying General Certificate Examination syllabus and examinations.

Contributions of SMP :-

- Books 1-5 form the main series of pupil's texts starting at the age of 11+ and leading to the O-level exam in SMP mathematics.
- Books 3T, 4 and 5 give a three years course to the same O- level examination.
- Advanced mathematics books 1-4 cover the syllabus for the A-level examination in SMP mathematics.
- SMP further mathematics is the shorter texts covering the various sections of the A - level.
- Two books for SMP Additional Mathematics at O - level

- Every book is accompanied by a teacher's guide.
- SMP texts are concomitant with modern mathematics syllabus of many boards.

By 1967 there was rider application of SMP texts than anticipated. They prepared a new series of books A -H which could serve as secondary school course starting at the age of 11+,suitable for preparing CSE examinations.

3.Nuffield Mathematics Project :-

The Nuffield Mathematics Project was set in 1964 by the Nuffield Foundation of Nuffield, England under the direction of Geoffery Mathews.

- Project was to develop new mathematics teaching for children.
- Project was designed for children of age 5 to 13.
- Aim :
  - To make activity of teaching mathematics lively in England.
  - Changed Arithmetic to Mathematics
  - Changed the teaching methods drastically for the 5 to 13 years old.
- Central notion - children must be set free to make their own discoveries and achieve understanding.

- They quote the old proverb ‘ I hear I forget, I see I remember, I do I understand’
- Emphasized development of new content materials appropriate to the laboratory method to teach elementary school mathematics.
- Project describes a motivational method, the method uses real life and environmental materials.
- This type of learning does not need much time for practice.

- Readily available materials are preferred
- The project provides a series of teacher's guide.
- The project made use of the theories and experiments of Jean Piaget.
- Emphasize child centred learning.
- Stress the need for interaction of the children in exploring situations.
- Nuffield approach helped pupils in topics where spatial awareness was involved.
- Study was conducted to check the assumption of Nuffield scheme then it is concluded that the difference in spatial ability between boys and girls increased favouring the boys.
  The title is **'I DO AND I UNDERSTAND'**

4.National Council for Educational Research and Training (NCERT):-

NCERT was established in India as an autonomous organization in September 1961 and is located at Sri Aurobindo Marg, New Delhi. It is concerned with research, instruction, and evaluation.

- The council runs 5 colleges of education at Ajmer, Bhopal, Bhubaneswar, Mysore and Shillong.
- These colleges conduct summer school cum correspondence in - service courses for secondary school teachers.

- The implementation of NPE made it necessary to review courses of studies in science and mathematics and also to bring out new text books.
- Besides, there were rapid developments in the different branches of science -pure and applied including mathematics.
- The NCERT was assigned the responsibility of developing the new curriculum and related curricular materials in line with new education policy to serve as models for states and the Union Territories to adopt.
- Appointed a General Advisory Board for science and mathematics under chairmanship of C. N. R Rao.
- Six writing teams were constituted by NCERT for developing instructional packages.

- They also developed additional instructional materials.
- Features of Text books developed by NCERT:
  - Able to develop critical power of analysis and reasoning.
  - Needs of different categories of children, especially those who have special aptitude and talents for studying mathematics are catered.
  - New concepts are introduced by means of simple examples.

5.National Curriculum Framework (NCF-2005):-

- NCERT was established in India as an autonomous organization in September 1961 and is located at Sri Aurobindo Marg, New Delhi. It is concerned with research, instruction, and evaluation.
- Observations on mathematics education by NCF-2005:
  - Developing children's abilities for mathematization is the main goal of mathematics education.
  - Narrow aim - to develop useful capabilities, particularly those relating to number operations, measurements, decimals and percentages.
  - Higher aim - to develop the child's resources to think and reason mathematically, to pursue assumptions to their logical conclusion and to handle abstraction.

Hence we need a curriculum that is,

1. Ambitious in the sense that it seeks to achieve the higher aim.
2. Coherent in the sense that it includes the variety of methods and skills in different branches.
3. Important in the sense that the students and teachers find it worth their time and energy addressing the problems.

Main objective behind NCF :

- Learn to enjoy mathematics.
- Pose and solve meaningful problems.
- Use abstractions to perceive relationships, to see structures, to reason out things and to argue the truth or falsity statements.
- Understand the basic structure of mathematics which offers a methodology for abstraction, structuration, and generalization.

NCF is for:

- Shifting focus of mathematics education from narrow to higher
- Engaging every student with a sense of success
- Changing methods of assessment to examine mathematization abilities rather than procedural knowledge
- Enriching teachers with variety of resources.

6.Kerala Curriculum Framework (KCF 2007):-

- Kerala Curriculum Framework (2007) based on the suggestion of NCF(2005), made ramarkable reforms in the school curriculum in Kerala
- Students learn for realizing social Justice, sustainable development, developing patriotism, scientific attitude, maintaining cultural uniqueness and becoming self reliant.
- Curriculum should be centred on social issues and the content must enhance the socially constructed knowledge to the level of critical response.

- Learning has to be based on principles of constructivism.
- Observe learning as an active mental process and building of knowledge.
- KCF analyses the causes of poor performance of students in mathematics and suggests some measures to overcome these situations.
- Higher aims of school mathematics :
  - Mathematization of thought process
  - Clarity of thought
  - Pursue assumptions to their logical conclusion
  - Handle abstractions
  - Ability and the attitude to formulate and solve problems.

2 Mark Question

1. Write two features of NCERT Curriculum?
2. Define curriculum?
3. Mention any two mathematics curriculum reforms?
4. Mention any four objectives of NCERT?

4 Mark Short Essay

1. Describe the logical and psychological approach in curriculum construction?
2. What is the role of Mathematics curriculum?
3. Explain NCF with special reference of mathematics curriculum?
4. Explain the steps in the development of mathematics curriculum?
5. Briefly explain any four types of curriculum?
6. What are the principle of curriculum construction and its organization?

10 Mark Essay

1. What are the principles of mathematics curriculum construction and organization?
2. What are the modern trends in curriculum construction? How far these principle observed in the construction of the curriculum in mathematics for the secondary school in our state?
3. Explain the steps of curriculum development. Given brief description of important Mathematics curriculum reforms?